



## Design of Refractive Index on the Basis of Desired Bend Loss, Propagation Mode and Dispersion in the W-Type Optical Fiber

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### ABSTRACT

Bend loss, propagation mode and dispersion are important factors in manufacture and use of optical fiber. These factors are function of refractive index and radiuses of core and clads. In this paper will be presented a new approach to determining of refractive index of core and clads on the basis of desired bending loss, propagation mode and dispersion in the interval 1250-1550 nm, and showing that each refractive index corresponds to sellmeier equation therefore designed fiber will be realized.

**Keywords:** *W-type optical fiber, bend loss, leakage loss, dispersion*

### 1. INTRODUCTION

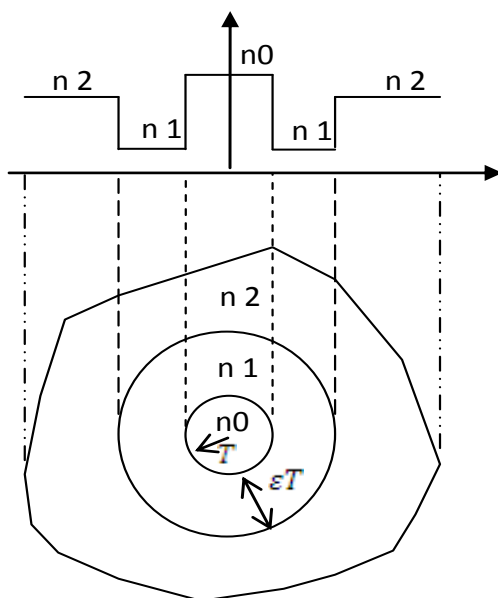
In 1974 Nishida & Kawakami [1] presented fundamental equations and evaluated leakage loss (in straight fiber) of W-type optical fiber. Several years later in 1982 leakage loss and bend loss in the other form was presented by Marcuse [2] and again in 1989 leakage loss and bend loss was studied by Vassallo [3]. A new method for bend loss evaluation in 1992 was presented by Renner [4] that this method is the approximation of other method that was presented by Faustini & Martini [5] in 1997.

In this paper with keeping of fiber parameters in desired levels and with guaranteeing of realizing of fiber manufacture, refractive index of core and clads design so that in band of 1310 (nm) to 1550 (nm) evaluated bend loss curve (on the basis of new form of Faustini & Martini method) and desired bend loss curve will be correspond.

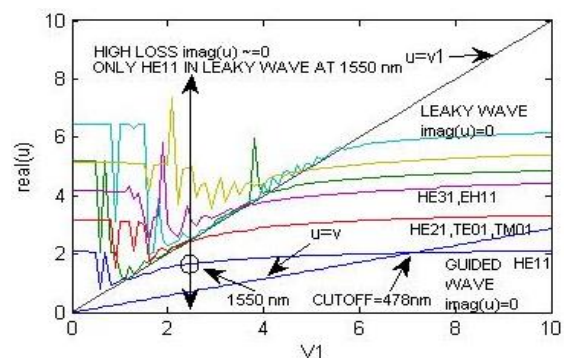
### 2. FUNDAMENTAL EQUATIONS OF W-TYPE OPTICAL FIBER

W-type refers to an optical fiber whose refractive index of layers is shown in Fig.1.If consider Hybrid mode[1]:

$$H_z = \pm y f(r) \sin n\theta, E_z = f(r) \cos n\theta, y = n_0 \left( \frac{\epsilon_0}{\mu_0} \right)^{\frac{1}{2}} \quad (1)$$



**Fig. 1. Cross section of W-type Optical fiber**  
Second clad can be irregular



**Fig. 2. Real part of transverse propagation constant in core for several mode with parameters: T=4.5µm, bn=.9997, an=0.99636, n0=1.4360, c=7**

sign" + " for  $HE_{nm}$  and "-" for  $EH_{nm}$  modes. By solving the Helmholtz equations in cylindrical coordinate and if Laser propagated in directions  $\mp z$  with  $e^{\pm z}$  factors  $f(r)$  is:

$$f(r) = p_j \left( \frac{ur}{T} \right) \quad r < T \quad \text{in core}$$

$$f(r) = qI_n\left(\frac{w_1 r}{T}\right) + sk_n\left(\frac{w_1 r}{T}\right) \quad T < r < (\varepsilon + 1)T \quad \text{in first clad (2)}$$

$$f(r) = hk_n\left(\frac{w_1 r}{T}\right) \quad (\varepsilon + 1)T < r \quad \text{in second clad}$$

Here T radius of core and (1+ε)T radius of first clad and u, w<sub>1</sub>, w consecutively transverse propagation constant in

$$u^2 + w^2 = V^2 = (n_0 k_0 T)^2 (1 - b_n^2), \quad a_n = \frac{n_1}{n_0}, \quad b_n = \frac{n_2}{n_0}, \quad k_0 = \omega(\varepsilon_0 \mu_0)^{\frac{1}{2}} = \frac{\omega}{c_0} = \frac{2\pi}{\lambda_0} \quad (3)$$

Here k<sub>0</sub> is the wave number in vacuum and V<sub>1</sub>, V linear normalized frequency in the first and second clad.

Therefore phase constant:  $\beta^2 = (n_0 k_0)^2 - \left(\frac{u}{T}\right)^2 = (a_n n_0 k_0)^2 - \left(\frac{w_1}{T}\right)^2 = (b_n n_0 k_0)^2 - \left(\frac{w}{T}\right)^2$

$$, V^2 = \frac{1-b_n^2}{1-a_n^2} V_1^2, \quad \lambda_0 = \frac{2\pi n_0 T \sqrt{1-a_n^2}}{V_1}$$

And finally by evaluating the fields and with the aid of boundary conditions, fundamental equation for HE<sub>n+1, m</sub> modes (n≥0) is:

$$F(u, w_1) = \epsilon(u, w_1, w, c) \quad (4)$$

Here : c = 1 + ε ,  $F(u, w_1) = \frac{j_n(u)}{u \cdot j_{n+1}(u)} - \frac{k_n(w_1)}{w_1 \cdot k_{n+1}(w_1)}$

$$\begin{aligned} &\epsilon(u, w_1, w, c) \\ &= X(w_1, c) \cdot W(u, w_1) \cdot K(w_1, w, c) \cdot X(w_1, c) \\ &= \frac{I_{n+1}(w_1) \cdot k_{n+1}(cw_1)}{k_{n+1}(w_1) \cdot I_{n+1}(cw_1)} \end{aligned}$$

$$2\alpha \left(\frac{\text{db}}{\text{m}}\right) = \text{Real} \left\{ \frac{4\pi\sigma k^2 \exp(-2\gamma b)}{\beta_0(\gamma^2 + \sigma^2) k_1^2(\gamma a)} \cdot \frac{1}{V_1^2(\lambda)} \right\}, \quad V_1(\lambda) = \frac{2\pi n_0 T \sqrt{1-a_n^2}}{\lambda} \quad (5)$$

$$k = \frac{u}{T}, \quad \gamma = j \frac{w_1}{T}, \quad \sigma = \frac{w}{T}, \quad b = (1 + \varepsilon)T, \quad a = T.$$

In the above relations  $k_0 = \frac{2\pi}{\lambda_0}$  is the wave number in vacuum, β<sub>0</sub> is the phase constant with extended first clad to infinity and k<sub>1</sub> is modified Bessel function. Fig. 3 shows leakage loss for some fiber parameter indicated in Fig. 2. In Fig. 3, you can see Leakages loss for normalize frequency V<sub>1</sub> ≤ 7.4. Leakages loss has positive values and for V<sub>1</sub> > 7.4 is down to zero (log(2α) → -∞). This means that fundamental mode (HE<sub>11</sub>) passes from leaky wave region to guided wave region in this normalized frequency (V<sub>1</sub> = 7.4, Fig. 2). This normalizes frequency (V<sub>1</sub> ≈ 7.4) is called cutoff frequency and its fixed value in two Figures 3, 2 shows correctness of evaluation. In Fig. 3 the amount of leakage loss for 1550 nm is about 10<sup>-5</sup>(db / m ) which is a negligible value.

the core, first clad, second clad and p, q, s, h are four constant that can be determined by boundary conditions. For u, w<sub>1</sub>, and w:

$$u^2 + w_1^2 = V_1^2 = (n_0 k_0 T)^2 (1 - a_n^2),$$

$$\begin{aligned} W(u, w_1) &= \frac{j_n(u)}{u \cdot j_{n+1}(u)} + \frac{I_n(w_1)}{w_1 \cdot I_{n+1}(w_1)}, \quad K(w_1, w, c) \\ &= \frac{k_n(cw_1)}{cw_1 k_{n+1}(cw_1)} - \frac{k_n(cw)}{cw k_{n+1}(cw)} \\ &= \frac{I_n(cw_1)}{cw_1 I_{n+1}(cw_1)} - \frac{k_n(cw)}{cw k_{n+1}(cw)} \end{aligned}$$

From the numerical solution of (2) the quantity of u (in general is a complex value but in reigns leaky wave and guided wave imaginary part of u approximate is zero) is obtained that is used in the next calculations. Fig.2 shows the amount of real (u) for an example.

### 3. LEAKAGE LOSS IN STRAIGHT FIBER

There are a few methods for the approximation of leakage loss, from Marcuse method [2]:

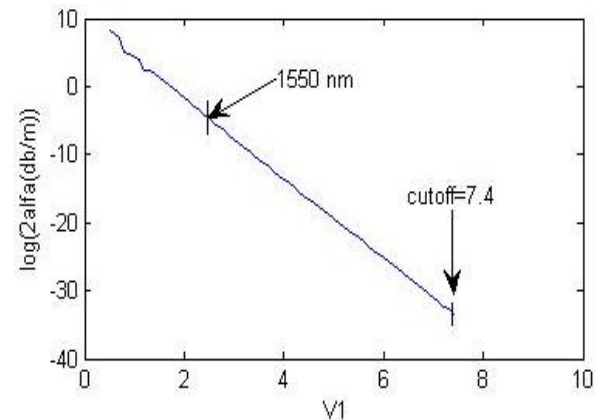
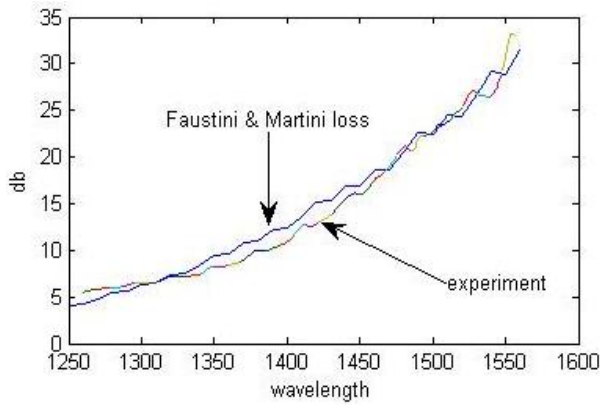


Fig. 3. Log of leakage loss (2(db/m)) for fundamental Mode with parameter: T=4.5μm, b<sub>n</sub>=.9997, a<sub>n</sub>=0.99636, n<sub>0</sub>=1.4360, c=7)



**Fig. 4. Compare between result of experiment and Faustini evaluation method for a Ring with parameters:  $T=4.5 \mu\text{m}$ ,  $b_n=0.9997$ ,  $a_n=0.99632$ ,  $n_0=1.4445$ ,  $c=7$ ,  $R=5\text{mm}$**

$$2\alpha = \text{Real} \left\{ \frac{-2k_p^2}{\beta V^2 k_1^2 (a\gamma)} \int_0^\infty \frac{\exp \left[ -a(\gamma^2 + \omega^2)^{\frac{1}{2}} \right]}{(\gamma^2 + \omega^2)^{\frac{1}{2}}} \cdot \frac{A_i[X_2(0, \omega)]}{B_i[X_2(a, \omega)]} \cdot \text{imag} [G_{23}(\omega)] d\omega \right\} \quad (6)$$

$$G_{23} = \frac{G_3(\omega) B_i[X_2(b, \omega)] - \frac{\partial}{\partial x} B_i[X_2(x, \omega)] \Big|_{x=b}}{\frac{\partial}{\partial x} A_i[X_2(x, \omega)] \Big|_{x=b} - G_3(\omega) A_i[X_2(b, \omega)]}, G_3 = \frac{\frac{\partial}{\partial x} B_i[X_3(x, \omega)] \Big|_{x=b} - j \frac{\partial}{\partial x} A_i[X_3(x, \omega)] \Big|_{x=b}}{B_i[X_3(b, \omega)] - j A_i[X_3(b, \omega)]}$$

$$X_q(x, \omega) = \left( \frac{R}{2k_0^2 n_q^2} \right)^{\frac{2}{3}} \left[ \beta^2 + \omega^2 - k_0^2 n_q^2 \left( 1 + \frac{2x}{R} \right) \right]$$

$$V = k_0 a n_0 \sqrt{1 - a_n^2}, k_p^2 = k_0^2 n_0^2 - \beta_0^2, \gamma^2 = \beta_0^2 - k_0^2 a_n^2 n_0^2$$

Here :  $q = 2, 3$ . Table 1 shows the parameters used in relations (6) for correspondence to Fig. 1 and fundamental relations (2). Fig.4 shows bend loss in terms of wavelength for a ring (360°) with a radius of 5mm that

#### 4. CUTOFF WAVELENGTH

In Fig. 2 the intersecting point of each curve of  $\text{real}(u)$  and line of  $u=V$  is a cutoff normalizes frequency for each mode and from  $\lambda_0 = \frac{2\pi n_0 T \sqrt{1 - a_n^2}}{V_1}$  can be determine the related wavelength. after the cutoff wavelength, each mode is in wave guided region and leakage loss will be zero.

#### 5. BENDING LOSS

There are several various methods for bend loss evaluation. But Faustini & Martini's [5] method with the new form is more suitable for computer simulation:

is plotted by relation (6). In Fig.4 although the two curves are close, they don't correspond.

In the continuation of this paper will try to find practical method to correspond them (experimented by MTS-8000 sys JDSU co.).

**Table1. parameter used in (6) for correspondence to Fig.1 & (4)**

Parameter in (6)	Parameter in (4) and Fig.1 And definition
$k_0$	wave number in vacuum
$\beta_0$ & $\beta$	phase constant for infinite first clad & finite first clad
$1 + 2x/R$	correcting bend coefficient for $n^2$
$a, b, k_p, \gamma$	$T, (1 + \epsilon)T, \frac{u}{T}, \pm j \frac{w_1}{T}$
$u, w_1, w$	propagation constant in core, first and second clad
$n_q$	refractive index in region $q=2,3$
$k_1$	modified Bessel function
$V$	linear normalize frequency in first clad
$n_2$ & $n_3$	$a_n n_0$ & $b_n n_0$ Refractive index in first and second clad

## 6. DISPERSION

If refractive index is determined by sellmeier relation material dispersion can be define as:

$$n_0^2(\lambda) = 1 + \frac{B_1 \lambda^2}{\lambda^2 - \lambda_1^2} + \frac{B_2 \lambda^2}{\lambda^2 - \lambda_2^2} + \frac{B_3 \lambda^2}{\lambda^2 - \lambda_3^2} \Rightarrow M = \frac{\lambda}{c} n''_0(\lambda) \quad (7)$$

Here:  $n''_0(\lambda) = \frac{\lambda n'_0(\lambda)[n_0^2(\lambda)+1]+n_0(\lambda)-n_0^3(\lambda)}{\lambda^2 n_0^2(\lambda)} - \frac{3\lambda^2 p(\lambda)n_0(\lambda)+\lambda^3 n_0(\lambda)p'(\lambda)-\lambda^3 p(\lambda)n'_0(\lambda)}{n_0^2(\lambda)}$

$$n'_0(\lambda) = \frac{n_0^2(\lambda) - 1}{\lambda n_0(\lambda)} - \frac{\lambda^3 p(\lambda)}{n_0(\lambda)}, \quad p(\lambda) = \frac{B_1}{(\lambda^2 - \lambda_1^2)^2} + \frac{B_2}{(\lambda^2 - \lambda_2^2)^2} + \frac{B_3}{(\lambda^2 - \lambda_3^2)^2}$$

Waveguide dispersion defines as:

$$M' = \frac{\lambda}{c} n''_{eff}(\lambda) \quad (8)$$

Here:  $n''_{eff}(\lambda) = \frac{n_0^2(\lambda) - n_{eff}^2(\lambda) + n_0(\lambda)n''_0(\lambda)}{n_{eff}(\lambda)} - \frac{[\lambda^2(u'^2 + uu'') + 4\lambda u u' + u^2]}{z \cdot n_{eff}(\lambda)}, z = 4\pi^2 T^2$

$$n'_{eff}(\lambda) = \frac{n_0(\lambda)n'_0(\lambda)}{n_{eff}(\lambda)} - \frac{uu'\lambda^2 + u^2\lambda}{z \cdot n_{eff}(\lambda)}, \quad n''_{eff}(\lambda) = n''_0(\lambda) - \frac{u^2\lambda^2}{4\pi^2 T^2} = n''_0(\lambda) - \frac{u^2(\lambda, n, m) \lambda^2}{z}$$

$u(\lambda, n, m)$  is transverse propagation constant in core that evaluated by the numerical method from (2) and  $n_0(\lambda)$  (sellmeier equation) and  $\lambda$  is wavelength of outer laser beam that differs from wavelength interrering optical fiber. therefore total dispersion:

$$D = M + M' \quad (9)$$

## 7. GROUP REFRACTIVE INDEX AND EFFECTIVE GROUP REFRACTIVE INDEX

These two items define as:

$$n_{og}(\lambda) = n_0(\lambda) - \lambda \frac{dn_0(\lambda)}{d\lambda} \quad (10)$$

$$n_{effg}(\lambda) = n_{eff}(\lambda) - \lambda \frac{dn_{eff}(\lambda)}{d\lambda}$$

## 8. DESIGNING EXAMPLE

In this section try to designing of single mode fiber with  $c = 7, T = 4.5 \mu m$  and flowing characteristics:

- A) Effective group refractive index in 1550 nm >1.462
- B) Total Dispersion close to zero in 1485 nm
- C) Total Dispersion in interval 1500-1550 nm <7 ps/nm.km

$$M' = \frac{\lambda}{c} n''_{eff}(\lambda), [u'(\lambda) \approx -4.3 \times 10^{-4}, u''(\lambda) \approx 0] \quad \lambda=1550$$

- D) Total Dispersion slope in 1500-1550 nm < .097 ps/nm<sup>2</sup>.km
- E) Cutoff wavelength <500 nm
- F) Bend loss for a ring with radius 5mm in interval 1260-1560 nm correspondence to Fig. 5.

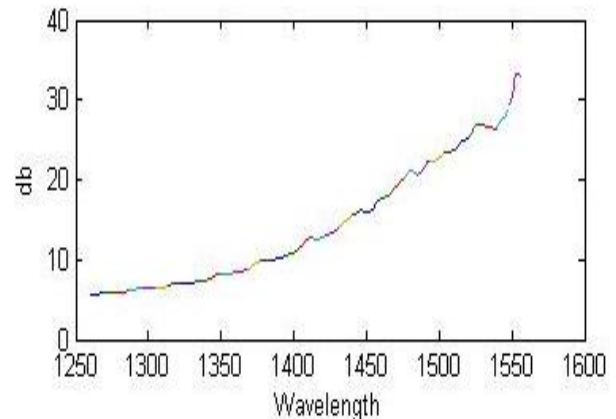


Fig. 5. Loss of a ring with parameters  $T=4.5 \mu m, c=7$  and radius  $R=5mm$  experimented by MTS-8000 system JDSU co.

Polarization mode dispersion, absorption and scattering are not involved in my design because these items depend on the length of the optical fiber (usually in km) and for a few centimeters they have negligible values. In the first step approximate of  $u', u'', n''_{eff}, M'$  from Fig. 2 about 1550 nm can be shown:

$$n_{\text{eff}}''(\lambda) \cong \frac{n_0'^2(\lambda) - n_{\text{eff}}'^2(\lambda) + n_0(\lambda)n_0''(\lambda)}{n_{\text{eff}}(\lambda)} - \frac{u^2(\lambda, m, n)}{z \cdot n_{\text{eff}}(\lambda)}, T=4.5\text{e-}6, b_n=.9997, a_n=0.99636, n_0=1.4360, c=7$$

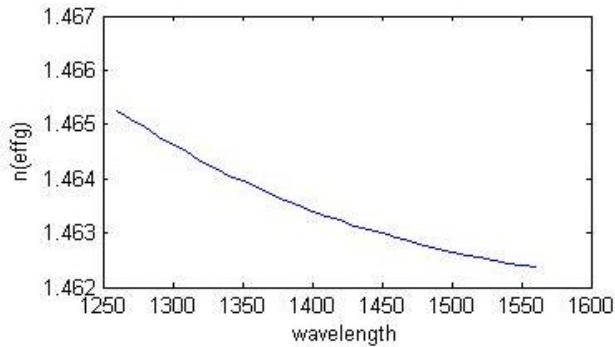


Fig. 6. Effective group refractive index with parameters  $\lambda_1=68.4043$  nm,  $\lambda_2=200$  nm,  $\lambda_3=10500$  nm,  $B_1=6961663, B_2=4079426, B_3=8974794$  for core

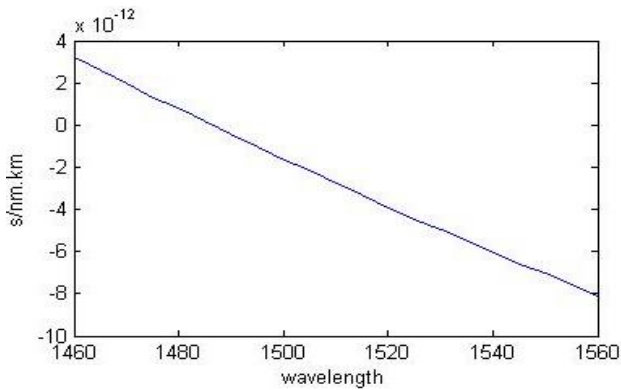


Fig. 7. total dispersion with parameters  $\lambda_1=68.4043$  nm,  $\lambda_2=200$  nm,  $\lambda_3=10500$  nm,  $B_1=6961663, B_2=4079426, B_3=8974794$  for core

Suppose that  $B_1, B_2, B_3$  for core are the same values for silica (in practice  $B_1, B_2, B_3$  may be different but here presentation of an evaluation method is desired). with a little change in  $\lambda_1, \lambda_2, \lambda_3$  for values  $\lambda_1=68.4043$  nm,  $\lambda_2=200$  nm,  $\lambda_3=10500$  nm, and from (7),(8),(9),(10) conditions A), B), C), D) will be realized ( Fig. 6, Fig. 7 ). For condition E) Fig. 8 shows a good correspondence between evaluated and desired curves and as shown in Fig. 2, Fig. 3 the cutoff wavelength is approximately:

$$V_1(\text{cutoff}) \approx 7.3, n_0 = 1.4345,$$

$$A_n = .99630,$$

$$T = 4.5\mu\text{m} \Rightarrow \lambda_0 = \frac{2\pi n_0 T \sqrt{1 - a_n^2}}{V_1} = 477.511 \text{ nm} < 500 \text{ nm}$$

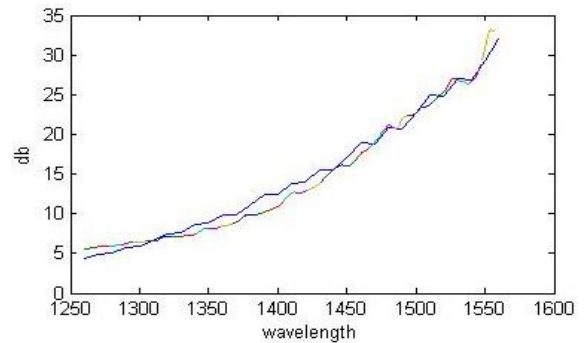


Fig. 8. A good correspondence between evaluated and desired curves with parameters:  $a_n=0.99630, n_0=1.4345$

For condition F) in a common plan we must several times plot the desired curve (Fig. 5) and the evaluated curve (for a fiber ring with a 5mm radius and the constant  $a_n$  and convenience  $n_0$  that two curves have at least one common point as shown in Fig. 9, Fig. 10). For example in Fig.9, Fig.10,  $n_0(1276) = n_0(1552) = 1.4245$  and  $n_0(1292) = n_0(1299) = n_0(1551) = 1.4345$  for constant  $a_n = .99632$ .

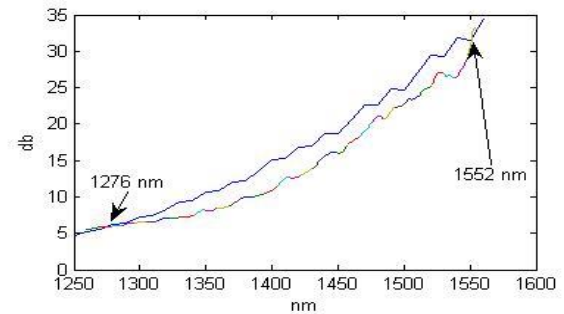


Fig. 9. Common point of desired curves and evaluated curves from Faustini method:  $a_n=0.99632, n_0(1276) = n_0(1552) = 1.4245$

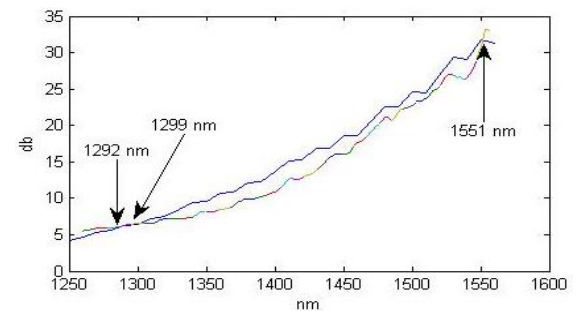
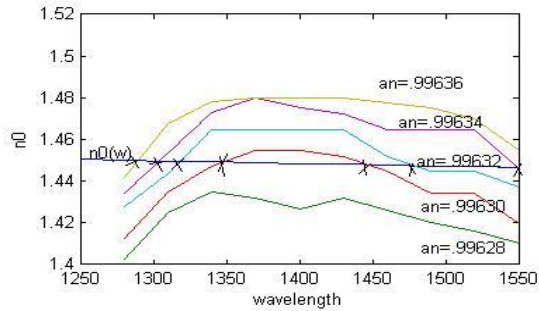


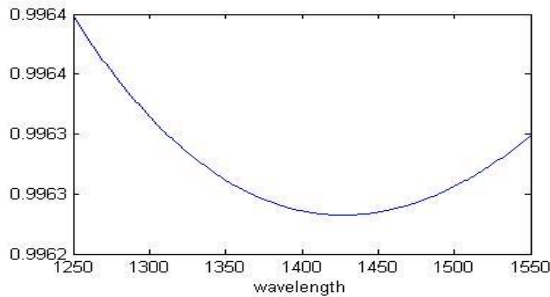
Fig. 10. Common point of desired curves and evaluated curves from Faustini method:  $a_n= 0.99632, n_0(1292) = n_0(1299) = n_0(1551) = 1.4345$ .

For several constant  $a_n$  curves in Fig.11, can be plotted by the change of  $n_0$ . On the other hand  $n_0(\lambda)$  may satisfy the Sellmeier equation. As a result the intersection of  $n_0(\lambda)$

and  $n_0$  curves (determined in Fig. 11 by "x" sign) which are plotted by the above manner shows that  $a_n$  is the function of  $\lambda$  and is not constant (for example in Fig. 11,  $a_n(1310\text{nm}) \approx .99634$ ). The precise plot of  $a_n(\lambda)$  shown in Fig. 12.



**Fig. 11.  $n_0$  plotted in two manner: first from sellmeier equation and other from crossing of desired and evaluated loss curves with constant  $a_n$  calculating for several times.**



**Fig. 12. Precise plot of  $a_n(\lambda)$ .**

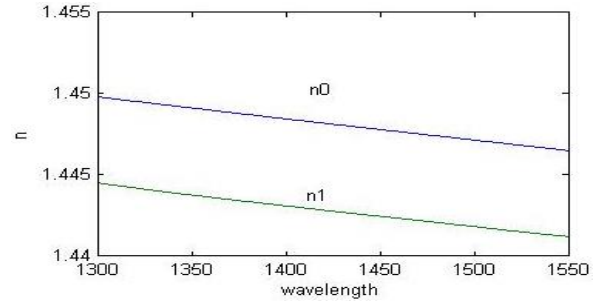
Now put  $n_1(\lambda) = a_n(\lambda) * n_0(\lambda)$ , Fig. 13 shows plotting of  $n_1(\lambda)$ ,  $n_0(\lambda)$ . In this Figure  $n_1(\lambda)$ ,  $n_0(\lambda)$  are parallel but not precisely. In Fig. 14 the difference between  $n_1(\lambda)$ ,  $n_0(\lambda)$  is plotted. If this difference did not exist, the curve shown in Fig. 5 would be a straight-line. Here if can be determine the values of  $B_1, B_2, B_3, \lambda_1, \lambda_2, \lambda_3$  for first clad that  $n_1(\lambda)$  in Fig.13, corresponds to sellmeier equation, it will be possible to make this optical fiber. For this purpose, because  $n_1(\lambda)$  in interval 1250-1550 nm near a straighten line by choosing six points and a little change in values we can determine the following values:

$$\lambda_1 = 60 \text{ nm}, \lambda_2 = 67.8 \text{ nm},$$

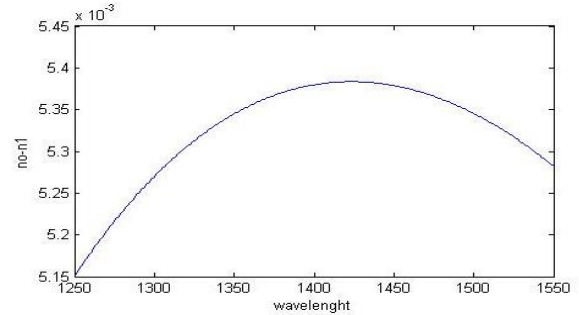
$$\lambda_3 = 11850 \text{ nm}, B_1 = -35.0676434, B_2 = 36.13653676,$$

$$B_3 = .4833681534$$

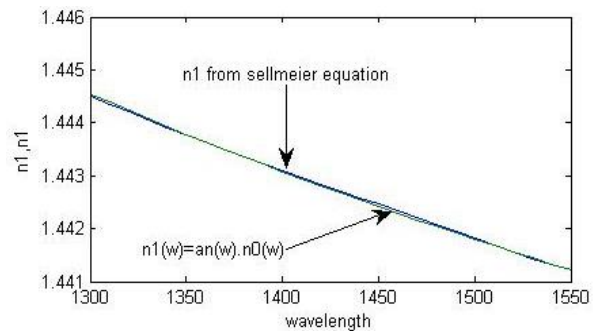
In Fig.15,  $n_1(\lambda)$  by sellmeier equation and relation  $n_1(\lambda) = a_n(\lambda) * n_0(\lambda)$  is plotted. As you can see these two curves approximately correspond (because for the manufacturing process, it is essential that refractive index be evaluated strictly by sellmeier equation). Finally if we put  $a_n(\lambda) = n_1(\lambda) / n_0(\lambda)$  the curve of bend loss from relations (6) (but with  $n_1, n_0$  function of  $\lambda$ ) approximately correspond to experimented curve (Fig. 16).



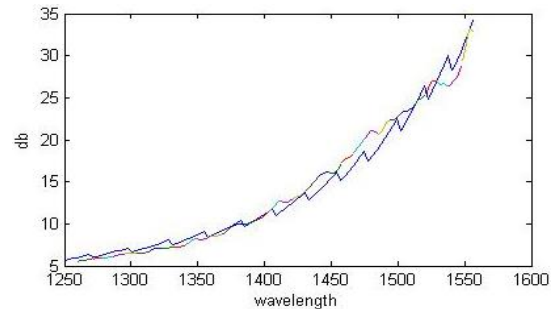
**Fig. 13. Plot of  $n_0(\lambda)$  and  $n_1(\lambda) = n_0(\lambda) * a_n(\lambda)$**



**Fig. 14. Plot of difference between  $n_0(\lambda)$  and  $n_1(\lambda)$**



**Fig. 15. . Plot of  $n_1(\lambda)$  in two manner: first from sellmeier equation and other From  $n_1(\lambda) = a_n(\lambda) * n_0(\lambda)$**



**Fig. 16. finally the best correspondence between evaluated and desired curves**

## 9. CONCLUSIONS

With this method can study the influence of radius and wavelength changes on the bend loss of optical fiber. With result of this paper we can control the bend loss and

slope of bend loss in each wavelength in 1250 -1550 nm interval and making attenuator and high pass filter for each wavelength in this interval.

## ACKNOWLEDGMENTS

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